Table 1 Error of time-predictor equation, $E(t) = T_{IBM} - T_{calc}$[a]

	W/C_DA = 1000		W/C_DA = 1500	
T_{IBM}, sec	h kft	$E(t)$, sec	h, kft	$E(t)$, sec
0	252.4	...	252.4	...
3	206.3	−0.005	...	...
6	160.0	−0.003	...	...
9	113.8	−0.001	...	...
12	68.7	+0.04	...	...
15	31.3	+0.15	...	...
18	9.7	+0.64	3.4	+0.19
18.4	...	...	0.0	+1.06
21	0.1	+1.98	...	...

[a] Based on β^{-1} = 23,650 ft; ρ_{SL} = 0.085 lb/ft³.

Upon multiplying through by $d\gamma$,

$$\frac{dV}{V} = \tan\gamma d\gamma + \frac{D\dot{\gamma}\, dt}{g \cos\gamma} + \frac{V\dot{V}}{gr}\, dt \tag{4}$$

$$= \tan\gamma\, d\gamma - \frac{D}{V}\, dt - \frac{V \sin\gamma}{r}\, dt \tag{5}$$

$$= \tan\gamma\, d\gamma - \frac{D}{V}\, dt - \frac{dr}{r} \tag{6}$$

Equation (6) is integrable and yields the desired result describing the exponential decay of the angular momentum which forms the basis of this paper.

Integrating Eq. (6) for a constant C_DA/m yields

$$\ln(r\, V \cos\gamma / r_0 V_0 \cos\gamma_0) = -M/2m \tag{7}$$

where the zero subscript indicates the initial conditions at time $t = 0$, and

$$M = M(t) = \int_{t_0}^{t} C_D A \rho\, V\, dt \tag{8}$$

is the air mass swept out by the effective missile cross-sectional area C_DA, where $\rho = \rho_{SL} \exp(-\beta h)$. Thus, the angular momentum is

$$P = P_0 \exp(-M/2m) \tag{9}$$

Time Predictor

Equation (9) can be equivalently stated as

$$d\,(r^2)/dt = 2P_0 \exp(-M/2m) \tan\gamma \tag{10}$$

Equation (8) can be written in the form

$$M(h,\gamma) = \int_{h_0}^{h} C_D A \rho_{SL} \exp(-\beta h) \frac{dh}{\sin\gamma} \tag{11}$$

With the assumptions that C_DA/m = constant and that $\sin\gamma$ varies slowly throughout the trajectory,

$$M/m = 2a[\exp(-\beta h) - \exp(-\beta h_0)] \tag{12}$$

where

$$a = -C_D A \rho_{SL} / 2\beta m \sin\gamma$$

Since γ is negative for entry, the approximated constant a is positive. Then Eq. (10) can be written

$$P_0 \tan\gamma\, dt = r\, dr \exp(-ae^{-\beta h_0}) \exp(ae^{-\beta h}) \tag{13}$$

Since

$$\exp(ae^{-\beta h}) = \sum_{n=0}^{\infty} \frac{(ae^{-\beta h})^n}{n!}$$

the right-hand side of Eq. (13) can be integrated termwise between the limits of h_0 and h, and the mean value theorem can be applied to the left-hand side. For application to ICBM re-entry trajectories, the mean value of $\tan\gamma$ can be chosen as the initial condition without introducing a significant error. Thus, setting the initial time $T_0 = 0$, the time-predictor equation becomes

$$T r_0 V_0 |\sin\gamma_0| = \exp[-ae^{-\beta h_0}] \left[\frac{r_0^2 - r^2}{2} + \frac{1}{\beta} \sum_{n=1}^{\infty} \frac{a^n e^{-n\beta h}}{n \cdot n!} \left(r + \frac{1}{n\beta}\right) - \frac{1}{\beta} \sum_{n=1}^{\infty} \frac{a^n e^{-n\beta h_0}}{n \cdot n!} \left(r_0 + \frac{1}{n\beta}\right) \right] \tag{14}$$

An examination of the convergence of Eq. (14) allows truncation after the first few terms, yielding a practical real-time solution to the time-predictor problem. For a given accuracy, the longest computation involves the summation of the series

$$\sum_{n=1}^{\infty} \frac{a^n e^{-n\beta h}}{n \cdot n!} \left(r + \frac{1}{n\beta}\right)$$

at the lower altitudes. For a computational accuracy of $10^{-4} r$, this series requires seven or eight terms for altitudes of 6000 ft or less. The exponential decay constant β and the density constant ρ_{SL} must be selected on the basis of the initial altitude h_0 for a best curve fit.

The equation was programed on a G-15 computer using the initial conditions obtained from an output of a set of IBM programs, which numerically integrated the equations of motion for constant W/C_DA on a 0.1-sec interval. The error of the time-predictor equation is negligible for $h > 50$ kft (Table 1). The initial extremely small errors can be attributed to the slight variations in the initial conditions caused by the five-digit input and calculation accuracy of the G-15 program.

In general, it can be concluded that the approximate solution for the time predictor is applicable to time prediction within a bounded region of a re-entry trajectory. The atmospheric constants ρ_{SL} and β should be selected in terms of the region of application for a best fit time prediction.

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Correlation for Maximum Pressure of "Hi-Lo" Igniter

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THE "Hi-Lo" igniter for solid propellant rockets is essentially a monovent rocket that burns pellets of propellant or pyrotechnic material.[1] Analytical prediction of the maximum pressure developed in the igniter by the pellet combustion products is complicated by the transient, short-term nature of the igniter pressurization. For example, a typical pressure-time curve is bell shaped with a zero pressure-to-

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Table 1 Properties of three ignition materials

Property	Ignition material M2	B	C
Composition	Nitroglycerine, nitrocellulose	Double-base + metal	Oxidizer + metal
Density, g/cm^3	1.65	2.21	1.67
ΔH_{comb}, cal/g	1080	1287	1500
Flame temp., °K			
Isochoric	3319	4000[a]	2500[a]
Isobaric	2712	3400[a]	2100[a]
Spec. heat ratio	1.224	1.18[a]	1.18[a]
Wt. fraction[b] of noncondensables	0.99	0.54	0.06
Burning rate[c]			
coefficient, a, $\times 10^3$	1.03	7.8	16
press. expt., n:	0.88	0.71	0.63
for P (psi) to:	8100	2060	2060
Impetus, 10^5 ft-lb/lbm	3.6	3.4[d]	1.6[d]

[a] Estimated.
[b] In products, based on room temperature.
[c] Rate $= aP^n$, in./sec, where P is pressure, psi.
[d] Experimentally determined, based on high-pressure bomb shots.

zero pressure time of less than 0.2 sec and a maximum pressure frequently lasting less than 0.01 sec. In the design of such igniters, often the only reliable way to obtain maximum-pressure data is to use trial-and-error testing. This paper describes a semiempirical method for predicting the maximum pressure of a Hi-Lo igniter with reliability satisfactory for design work.

Experimental

A schematic drawing of the igniter tested[1] is shown in Fig. 1. The essential features are: a small, cylindrical-shaped combustion chamber; a DeLaval nozzle whose throat area can be varied between shots; a ceramic-coated screen between the combustion chamber and the entrance to the igniter nozzle (to prevent igniter pellet slivers from blowing out during the shot); and electrical connectors for an initiator device used to ignite the pellets. A fast-response pressure gage was fitted into the combustion chamber of the igniter.

Tests were made using three different ignition material compositions. The general composition and some of the properties of these materials are given in Table 1. The M2 material was developed for use as a gun propellant.[2] The exact compositions of ignition materials B and C are classified (B was specially developed for the Naval Ordnance Laboratory ignition work, and C is commonly used in rocket type igniters[1]). With these three compositions, it was possible to evaluate the igniter performance for ignition materials that produce combustion products that are 1) mostly condensable, 2) partially condensable, and 3) mostly noncondensable, respectively. Two pellet geometries were used: solid cylinders (all three materials) and cylinders with a longitudinal center perforation (M2 material only). Pellet diameters ranged from 0.160 to 0.475 cm, with lengths from 0.475 to 0.953 cm and length-to-diameter ratios from 2 to 4.

Tests were made for each composition and pellet geometry in which one or more of the following parameters was varied: 1) weight of ignition material (1.0 to 10.0 g), 2) combustion chamber volume (10.3 to 28.1 cm^3), and 3) nozzle throat area (0.020 to 0.493 cm^2). About 500 tests were used in the correlation study reported here; reproducibility, based on igniter pressure-time histories for three or more identical shots, was good (e.g., the maximum igniter pressures for material B were reproducible to within $\pm 5\%$).

Correlation of the Igniter Maximum Pressure

A semiempirical correlation of the Hi-Lo igniter maximum pressure was developed from the test data. It was observed that, when the igniter shot conditions are such that the pellet consumption rate is low (e.g., when using very large-sized pellets and a relatively large nozzle throat area), the pressurization of the igniter is very similar to that of a highly regressive rocket. At the other extreme, when the consumption rate is very rapid (e.g., when using very small pellets and a relatively small throat area), the pressurization approximates that of a closed bomb (the burnout of the pellets corresponds to the attainment of the bomb maximum pressure). The shot conditions normally used in operational igniters result in pellet consumption rates intermediate between these two extremes and in igniter maximum pressures ranging between about 70 and 350 atm.

It was found that the igniter maximum pressure could best be correlated by

$$(P_c)_{\max} = a(K\Delta)^b \tag{1}$$

where $(P_c)_{\max}$ is the igniter maximum pressure; K is the ratio of the initial pellet surface area (including hole area for perforated pellets) to the igniter nozzle throat area; Δ is the packing density of the shot, i.e., the ratio of the initial weight of ignition material to the initial free volume (combustion chamber volume minus pellet volume); and a and b are constants related to the density, discharge coefficient, and burning-rate properties of the ignition material. The term Δ is analogous to a packing-density term used in closed-bomb pressure correlations, and the K is characteristic of rocket systems.[3]

The test data for the three materials are presented in Fig. 2; each plotted point represents an average of two or more shots. The equation constants a and b for each material were determined from a least-squares fits to give the solid lines. The dashed lines represent standard errors-of-estimate limits of ± 2; that is, 95% of the data would fall between the dashed lines for normal Gaussian distribution. It is seen

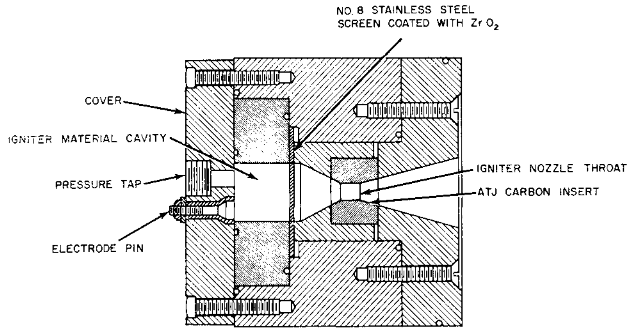

Fig. 1 Schematic drawing of the Hi-Lo igniter.

that, for the Hi-Lo igniter design and pellet geometries used, Eq. (1) worked equally well for ignition materials varying widely in thermodynamic and ballistic properties and for pressure test data resulting from about a three-fold range of igniter and pellet sizes.

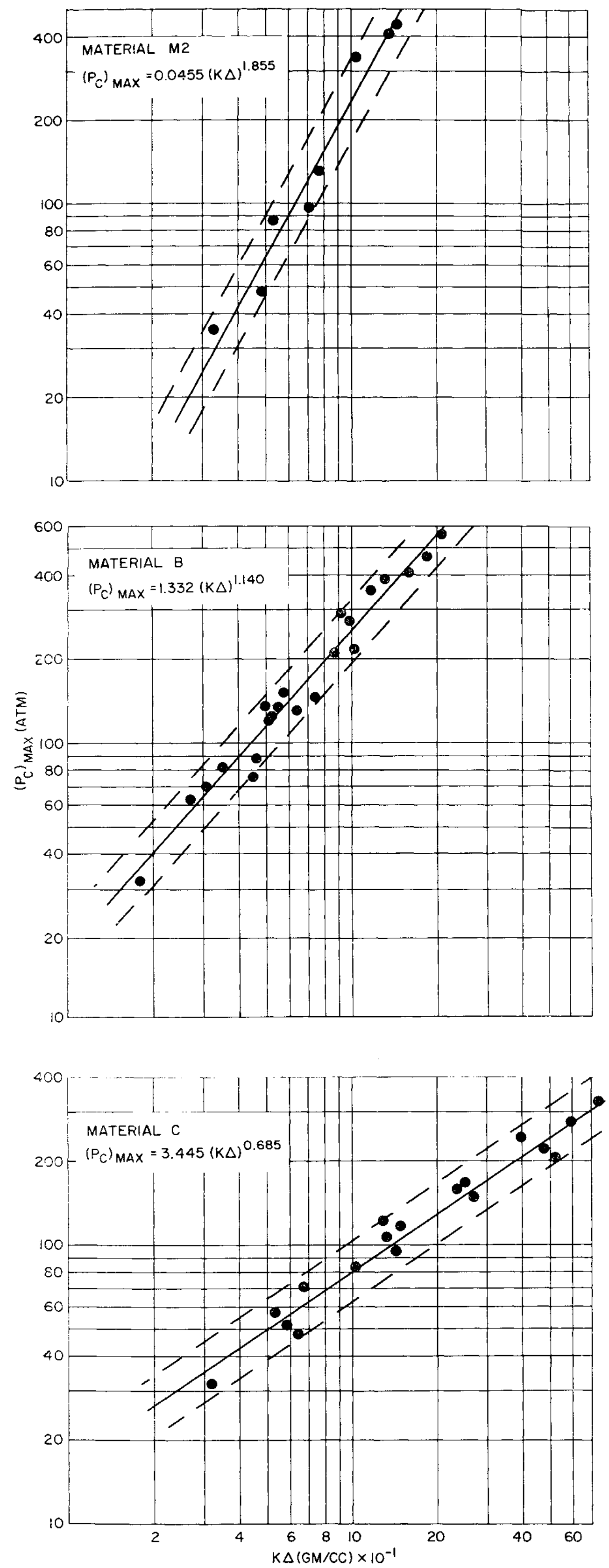

Fig. 2 Plots of igniter maximum pressure vs $K\Delta$ for three ignition materials.

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Heat Transfer in the Vicinity of Two-Dimensional Protuberances

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Nomenclature

h	=	local heat-transfer coefficient
h_0	=	local, flat plate, heat-transfer coefficient
L	=	distance from protuberance
M	=	Mach number
Re	=	local Reynolds number
Y	=	height of protuberance
X	=	wetted distance
δ	=	boundary-layer thickness
θ	=	angle of leading edge of protuberance

Introduction

BERTRAM'S[1] work in the hypersonic range clearly showed that an empirical equation could be written to define heat-transfer coefficients near protuberances. However, a literature survey failed to yield an analytical approach for calculating heat-transfer coefficients in the vicinity of protuberances in supersonic flow. The present work was performed to develop working equations for the latter purpose. Dimensional analysis was applied to the test data of Burbank[2] to obtain an empirical equation for the heat-transfer coefficient in each of the three regimes; upstream, immediately behind, and downstream of the protuberance.

Burbank's[2] experiments were conducted at freestream Mach numbers of 2.65, 3.51, and 4.44, Reynolds number per foot from 1.3×10^6 to 4.7×10^6, and boundary-layer thickness from 0.7 to 6.0 in. The 6.0-in. boundary layer normally exists at the wall of the test section of the tunnel. The smaller boundary layers, 0.7 and 1.5 in., were achieved by inserting a flat plate in the test section and then fixing the protuberance at the wetted length of flat plate required to give the desired

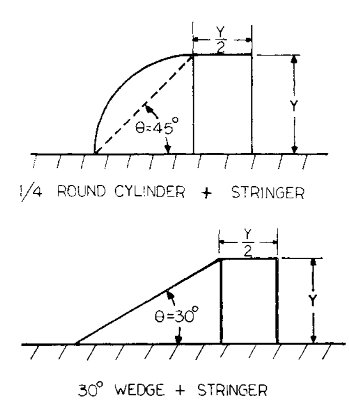

Fig. 1 Protuberances with "swept" leading edges placed in front of basic stringer (Y = 2 or 4 in.).

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